

Multilevel Modelling for Environmental Applications

Modelli multilivello per le applicazioni ambientali

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Riassunto: I modelli multilivello sono strumenti di uso corrente per l'analisi di dati osservazionali nelle scienze sociali. L'esempio principe è offerto dallo studio del rendimento scolastico, dove le unità statistiche (gli studenti) possono essere raggruppate in classi, che a loro volta possono essere raggruppate in scuole, distretti scolastici e così via. Tali raggruppamenti annidati danno luogo alla natura multilivello dei dati, caratterizzata dalla possibile elevata somiglianza delle unità statistiche all'interno dei medesimi gruppi. Di tale natura si deve tenere opportunamente conto per ottenere procedure statistiche corrette. Questo lavoro discute il ruolo dei modelli multilivello nelle applicazioni ambientali. I principi di base della modellazione multilivello verranno richiamati utilizzando un'applicazione a dati sul cancro della pelle. Verranno quindi introdotti i modelli multilivello non lineari, che risultano particolarmente rilevanti nelle applicazioni ambientali. Infine, verrà brevemente trattata la modellazione multilivello multivariata.

Keywords: Multilevel model, Hierarchical model.

1. Background

In a paper modelling teaching styles, Aitkin *et al.* (1981) demonstrated the potential for incorporating correctly structured random effects in an unbalanced observational study design. Much of the subsequent development of this analytical approach has taken place within this application area, and tends to be referred to as multilevel modelling. This reflects the way observations have been collected within a nested structure, but it should be noted for example that longitudinal data can be considered in this framework (where repeat observations on an individual can be considered at one level and the observed individual can be considered at the next level), and also that other developments deal with cross-classifications. There is a plentiful "multilevel modelling" literature within the social sciences, relatively recent textbooks include Snijders and Bosker (1999) and Hox (2002).

Before departing from the social sciences, it may be of some interest to note some applications which, it could be argued, touch on some important environmental considerations. For example, in an educational context, Wold *et al.* (2004) used a multilevel model to consider the effect of teacher smoking on pupil behaviour. Using this approach they were able to identify elements of national as well as school policy on smoking behaviour. Another "environmental" issue relates to recycling; by using a multilevel model Guerin *et al.* (2001) were able to examine both individual factors and institutional factors (local government activities) in determining recycling behaviour, finding that the former was a stronger predictor of recycling behaviour. There are numerous other examples, many such as Boslaugh *et al.* (2004); van Lenthe *et al.* (2005) concerning attitudes to transport,

which have been carried out at the interface between social science and environmental interests .

In this paper, attention will focus on applications from the natural sciences. Firstly, the basic structure of a multilevel model will be reviewed and illustrated with reference to data on Ultraviolet B radiation and melanoma rates. Secondly, given the importance of non-linear models; a recently published meta-analysis on benthic oxygen demand in estuarine systems will be explored. In this application, it is possible to determine population level distributions for the three parameters of interest, and use this information to estimate specific parameter distributions for an individual estuarine system. Finally, multivariate multilevel models will be considered which have potential in many environmental applications. In these models, it is possible to borrow strength across related variables as well as between related individuals within a nested structure.

2. Basic structure of a multilevel model

Multilevel models have been described as ordinary regression models but with a model superimposed on the regression parameters themselves (Gelman, 2005). In order to develop the notation, firstly assume a standard linear model as follows:

$$y_i = \beta_0 + \beta_1 x_j + \epsilon_j \quad (1)$$

where $E(\epsilon_j)$ is assumed zero and $var(\epsilon_j) = \sigma_\epsilon^2$. Various different notations exist when expanding this into a multilevel setting. As stated above, much work relating to “multilevel models” has been carried out in the educational sector where the following notation is commonly used (Goldstein, 2003). Assuming a single nested layer with i denoting the individual (such as a pupil) and j denoting the first layer in the nesting structure (the class), a simple multilevel model can be specified as:

$$y_{ij} = \beta_{0j} + \beta_{1j} x_{ij} + \epsilon_{ij} \quad (2)$$

but in equation (2), the slope and intercept are now assumed to be random variables. One way of considering these is by expanding the notation such that $\beta_{0j} = \beta_0 + \nu_{0j}$ and $\beta_{1j} = \beta_1 + \nu_{1j}$ where ν_{0j} and ν_{1j} are random effects reflecting individual level contributions to the intercept and slope respectively within these nests. According to this notation, β_0 and β_1 reflect the upper level of nesting, that is the class contributions to the intercept and slope respectively. It is common to assume that the random effects ν_{0j} and ν_{1j} are normally distributed with zero mean, i.e. $E(\nu_{0j}) = E(\nu_{1j}) = 0$, and where $var(\nu_{0j}) = \sigma_{\nu_0}^2$, $var(\nu_{1j}) = \sigma_{\nu_1}^2$ and $cov(\nu_{0j}, \nu_{1j}) = \sigma_{\nu_{01}}$. Whilst $var(\epsilon_j) = \sigma_\epsilon^2$, clearly much of the variation will have been partitioned within the random effects. This nesting structure can be extended to further levels if necessary.

A wide range of model fitting procedures are available for these models, and there is a reasonable range of software which incorporates various alternatives. Illustrations given here will either be based on routines available within the R software (R Development Core Team, 2004), or by methods based on MCMC using the WinBUGs software (Spiegelhalter *et al.*, 1998). Nevertheless, to supply some context to the multilevel field, reference is made firstly to Goldstein (2003), who considers a number of procedures based on Iterative Generalised Least Squares (IGLS). This maximum likelihood procedure was established by Goldstein and Rasbash (1992). However, full maximum likelihood procedures, especially for small samples yield biased estimators by generally underestimating the vari-

ance components. Most multilevel modelling procedures are therefore based on some approximation to maximum likelihood estimators. Goldstein (1989) proposed Restricted Iterative Generalised Least Squares (RIGLS) which yield Restricted / Residual Maximum Likelihood (REML) estimators. Lindstrom and Bates (1988) also proposed REML routines, which they fitted using a combination of the Expectation-Maximisation (EM) and Newton-Raphson algorithms; a number of the former steps being used to provide suitable starting estimates for the latter. It is possible to consider REML as “integrating out” the fixed effects (Pinheiro and Bates, 2000) which complicates model comparison somewhat. Nevertheless, these latter procedures are currently available in the R software (Pinheiro *et al.*, 2004).

Different approximations are used for non-normal data, Goldstein (1991) suggested a Marginal Quasi-Likelihood (MQL) approach. However, Schall (1991) suggested a Penalised Quasi-likelihood (PQL) which has been made available in the R software by Venables and Ripley (2002).

Given the definition of a multilevel model above, whereby the “fixed” effects parameters are considered in conjunction with random effects structured according to the nesting, it is clear that these models can be very naturally considered as Bayesian hierarchical models where all the parameters are assumed random. There is a particularly plentiful literature in this area. An application-focussed introduction can be found in Congdon (2001) who describes multilevel models within this framework and has provided several worked along with WinBUGs code.

It should be noted, despite widespread adoption of multilevel models in many application areas such as the social sciences, research is still ongoing in relation a number of the fundamental statistical and computational properties. For example, Lee and Nelder (1996) proposed the h -likelihood. This relaxes the assumption of normal random effects but has yet to be fully developed within the multilevel modelling framework. In a Bayesian context, Browne and Draper (2000) compared methods for multilevel logistic regression and proposed the use of an adaptive hybrid Metropolis-Gibbs sampling regime (made available in the MLWin software) which gave better results in simulation studies than the Adaptive Rejection Metropolis Sampler.

Whilst there is clearly scope for ongoing development of the methodology, particularly in relation to nested observational data, it is worth noting the long tradition of the use of multilevel models within the social sciences.

3. Illustration of a prototypical multilevel model

A convenient illustration of a prototypical multilevel modelling problem can be made with reference to melanoma rates in nine European nations and a possible association with UVB exposure. These data, analysed by Langford *et al.* (1998), are a particularly well-rehearsed example and serve to illustrate the importance of using multilevel models with nested observational data.

Data are provided on the number of reported cases of melanoma within 355 counties, which in turn are nested within 79 regions, which in turn are nested within the nine nations. Covariate information about UVB exposure is also provided. In the form in which the data have been made available this covariate is standardised. The data are illustrated in Figure 1 where the importance of the nation-level grouping is immediately clear. It should also be noted that there had been some interest in this particular application as to whether it is

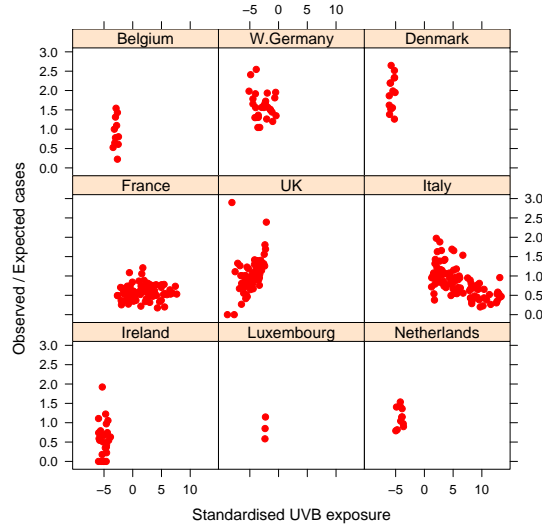


Figure 1: Standardised melanoma rate for nine European countries plotted against standardised UVB exposure

necessary to model UVB exposure with a quadratic term (Langford *et al.*, 1998).

These data can be modelled by some form of the generalised linear model (GLM). Assuming that the $n = 1, \dots, 355$ melanoma counts in each county to be denoted by $\mathbf{y} = (y_1, \dots, y_n)$ and the expected melanoma counts (offset) denoted by $\mathbf{o} = (o_1, \dots, o_n)$ it is possible to model the SMR in a three part specification with y_i assumed independent random variables from the Poisson distribution (an exponential family distribution) such that $E(\mathbf{y}) = \mu \mathbf{o}$. This expected response is related to a linear predictor, η , by means of a link function $g(\cdot)$ such that $\eta = g(\mu)$; in this case log links are used. Finally, the systematic part of the GLM η is modelled as a linear function. At this stage, it is instructive to ignore the multilevel structure of the data and to fit a simple GLM (Nelder and Wedderburn, 1972). This initial model therefore has a linear predictor:

$$\eta_i = \beta_0 + \beta_1 x_i \quad (3)$$

given covariates x_i , in this case the standardised values for UVB exposure. It is possible to include β_2 as the coefficient of the quadratic term for UVB exposure. Fitting these simple models gives either an intercept, $\hat{\beta}_0 = -0.0701$ and slope relating melanoma rates to UVB exposure $\hat{\beta}_1 = -0.0572$, or where a quadratic term is added for UVB exposure the model is estimated as $\hat{\beta}_0 = -0.0833$ with $\hat{\beta}_1 = 0.0584$ and $\hat{\beta}_2 = 0.0007$. Given deviance for the null model of 2357.30 and deviance of the UVB model of 1852.50 it can be seen that some improvement in fit is seen for the inclusion of a covariate. However, the deviance for the quadratic model is 1850.90 which seems to be a relatively negligible improvement in fit for the inclusion of the quadratic term.

Nevertheless, with deviance of around 1850 on just over 350 degrees of freedom it is very clear that considerable over-dispersion remains. There exist a number of well established methods for addressing such over-dispersion, for example the negative binomial model (Lawless, 1987) or the Poisson-log Normal model (Hinde, 1982). Therefore, given the multilevel nature of the data, it is necessary to structure these random effects in re-

lation to either nested intercepts or nested slopes. In other words the model to be used is:

$$\eta_{ij} = \beta_{0j} + \beta_{1j}x_{ij} \quad (4)$$

where either β_{0j} or β_{1j} , or both, can be assigned a multilevel structure by the inclusion of nested random effects at some combination of nation or region level. Formula (4) is therefore a prototypical multilevel GLMM for the melanoma data. As described above, these models can be very easily fitted by PQL (Venables and Ripley, 2002).

Table 1 provides results from fitting five models, with nation (N) level random intercepts (1), nation level random intercepts and slopes (2), nation and region (R) level random intercepts (3), nation and region level random intercepts with nation level random slopes (4) and finally nation and region level random intercepts with nation and region level random slopes (5). These results can be contrasted with the simple GLM reported earlier, for example without random effects the intercept $\hat{\beta}_0$ was estimated as -0.0701 ; with the addition of national random effects nested on the intercept $\hat{\beta}_0$ has been estimated as -0.0408 and with the addition of a regional within national random effects nested on the intercept $\hat{\beta}_0$ has been estimated as -0.0618 . Comparing results from table 1 with the earlier GLM also demonstrates how estimates of the slope $\hat{\beta}_1$ vary. The reason for these different fixed estimates are also demonstrated within table 1. The overdispersion within these data is partitioned within the nests, and the weight of evidence between nations with few regions and nations with many regions is balanced. It should be noted that these models are essentially log-linear models for the SMR, the standard deviation associated with each of the random effects is therefore particularly large relative to the fixed effects. example, from model 2 for Italy we would have an intercept $\hat{\beta}_{0 \text{ Italy}}$ of 0.0153 and a slope $\hat{\beta}_{1 \text{ Italy}}$ of 0.0139 ; for the UK we have an intercept $\hat{\beta}_{0 \text{ UK}}$ of -0.0622 and a slope $\hat{\beta}_{1 \text{ UK}}$ of -0.0565 . What makes these differences so noteworthy is that the basic model is unchanged; it relates melanoma rate to an intercept and a covariate. However, by structuring random effects which reflect possible nesting in the data it is possible to alter the estimates of the parameters. In effect, the multilevel model is suggesting that the melanoma rates differ substantively between regions and nations. Further, by considering national level random effects, it can be seen that the relationship between UVB and melanoma rate varies by country.

Model criticism remains of great interest. In addition to the monographs referred to earlier Hodges (1998) made a number of proposals to deal with specific considerations of these kinds of models. Nevertheless, it is possible to review the residuals within this multilevel structure. Here, it is possible to examine residuals within the national level nesting. As described earlier, there had been some previous interest as to the necessity of a

| Model | Intercept | UVB | $s.d.(\hat{\beta}_{0 \text{ N}})$ | $s.d.(\hat{\beta}_{0 \text{ R}})$ | $s.d.(\hat{\beta}_{1 \text{ N}})$ | $s.d.(\hat{\beta}_{1 \text{ R}})$ |
|-------------------------------------------------|-----------|-----------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| 1 <i>Intercept_N</i> | -0.0408 | -0.0265 | 0.3565 | | | |
| 2 <i>Intercept_NSlope_N</i> | -0.0408 | -0.0265 | 0.2566 | | 0.2328 | |
| 3 <i>Intercept_R</i> | -0.0618 | -0.0314 | 0.3635 | 0.2101 | | |
| 4 <i>Intercept_RSlope_N</i> | -0.0618 | -0.0314 | 0.2502 | 0.2371 | 0.2010 | |
| 5 <i>Intercept_RSlope_R</i> | -0.0618 | -0.0314 | 0.2503 | 0.2371 | 0.2010 | 0.1714 |

Table 1: Estimates of fixed effects, and standard deviation of random effects associated with various models fit to the melanoma data

quadratic term for ultraviolet exposure. Nation by nation residuals are presented from four models in Figure 2. “Random intercepts and UVB” corresponds to model 1, i.e. national level random intercepts and UVB; “Random intercepts and quadratic term” corresponds to model 1 with the addition of a quadratic term for UVB (i.e. national level random intercepts and UVB with UVB²; “Random intercepts and random slopes” corresponds to model 2 (national level random intercept and random slopes); finally “Null model” corresponds to model 1 without any covariates (null model with national level random intercepts only).

It is straightforward to assess these models by Bayesian methodology. Corresponding to model 1 earlier, the “fixed” effects β_0 and β_1 were assigned non-informative normal priors. The county level random effects ν_{ij} had nation specific hyper-priors for their mean and variance of $Normal(0, 10^6)$ and $Gamma(0.001, 0.001)$ respectively. It is possible to contrast these with models having county level random effects drawn from a common prior, i.e. county level random effects $\sim N(0, 10^6)$ (Global Random effects) as well as a model having no covariates (National random effects without UVB). The Deviance Information Criteria (DIC) has been proposed by Spiegelhalter *et al.* (2002) as a model fit criteria for these models. As with AIC and BIC the aim is to penalise more complex models, and it should be noted that a lower DIC suggests a better fitting model. Examining DIC diagnostics suggest that the most important variation within these data is between countries. Results are given in Table 2 in which \bar{D} refers to the posterior mean of the deviance, \hat{D} refers to the deviance at the posterior mean of the various model parameters, pD is derived from these values and is intended as a penalty term penalising more complex models. DIC appears to be lowest when national level random effects are included and the covariate term is excluded.

Clearly, there is much scope for examining these data in considerable detail. What is very clear from the above illustration is that the relationship between UVB and melanoma rates is very sensitive to the way in which the nested structuring of the data is handled. Clearly, multilevel modelling cannot provide a panacea to the ecological fallacy (Robinson, 1950), and we cannot assume that relationships seen in aggregate data are the same for the individuals being observed within that data. Nevertheless, it is certainly possible to improve the predictive ability of models by carrying out an analysis incorporating information on the nesting structure under which the data has been collected. Where the data have not been collected under a design controlled by the investigator, multilevel models may therefore act to retrospectively block a number of important confounding factors.

multilevel models can be seen to deal with unbalanced observational study design. In doing so, the estimate of important fixed effects is altered, in this case the evidence for the relationship between UVB and melanoma rates is substantially qualified, there are clearly significant differences in melanoma rates between these nine countries which need to be adequately modelled. However, multilevel models form a subset of hierarchical models where shrinkage of estimators is of particular interest. For example, Coull *et al.* (2003)

| | \bar{D} | \hat{D} | pD | DIC |
|-------------------------------------|-----------|-----------|---------|----------|
| Global random effects | 1946.310 | 1712.540 | 233.770 | 180.080 |
| National random effects | 1955.700 | 1616.850 | 338.847 | 2294.550 |
| National random effects without UVB | 1952.640 | 1796.490 | 156.156 | 2108.800 |

Table 2: Model fitting diagnostics for Bayesian multilevel models

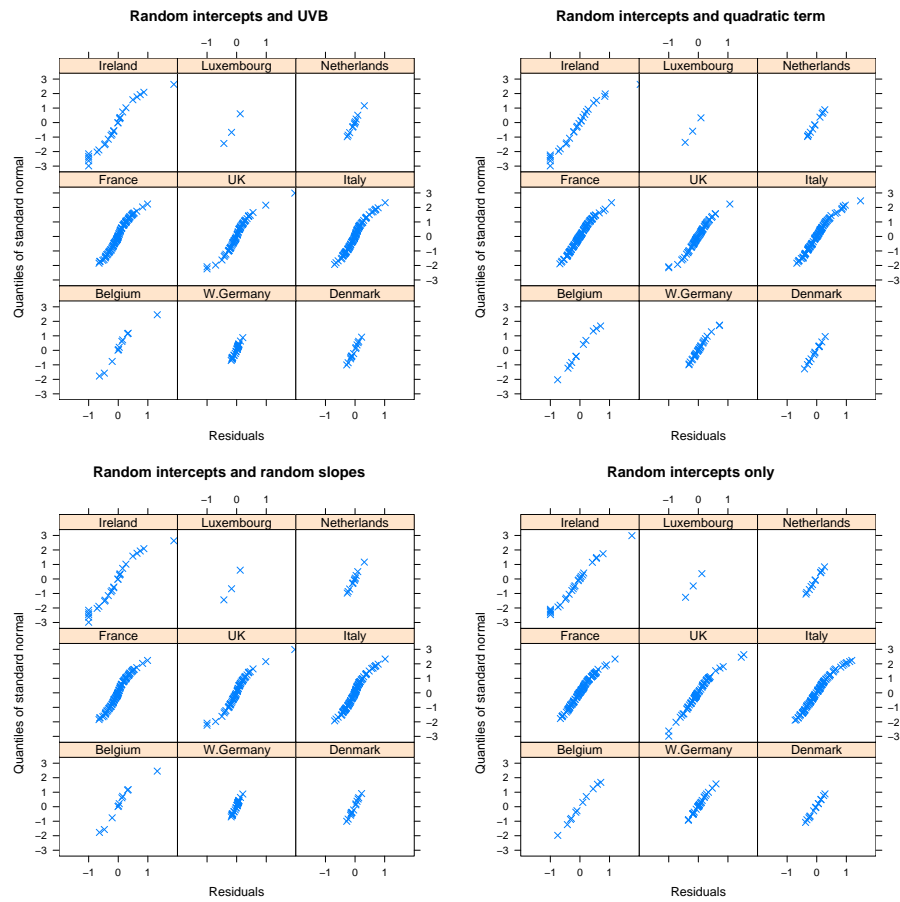


Figure 2: Quartile-Quartile plots of residuals from four models fit to the melanoma data; models are described in the text

used a multilevel analysis to share strength across studies (meta-analysis) in relation to the effects of intra-uterine mercury exposure. In the next section, we will consider models for benthic oxygen demand (BOD) in estuarine systems. These non-linear models have three parameters to be estimated from three data points. By using a multilevel structure it is possible to borrow strength in an appropriate manner from published analyses of other systems and obtain plausible estimates for a single system.

4. Non-linear multilevel models

Non-linear models are important in many of the physical and natural sciences for a variety of reasons. In addition to parsimony and the possibility that the model may be valid beyond the range of the data they are particularly useful where there is existing knowledge regarding a mechanistic or semi-mechanistic relationship within the data (Bates and Watts, 1988). An illustration will be given here in relation to a published meta-analysis on BOD in 34 estuarine systems reported in the literature over a period of nearly 3 decades. In reporting the meta-analysis, Borsuk *et al.* (2001) explain the derivation of a mechanistic model. However, for a single system these models are over-parameterised. The model for BOD is given as:

$$BOD = \alpha \left(\frac{L_C}{1 + \kappa L_C d} \right)^\beta \quad (5)$$

where L_C denotes Carbon Loading and d denotes the depth of the system. A trivial way of fitting such a model would be by means of conventional non-linear modelling.

This model could be fit with with residual standard error of 2.885 on 63 degrees of freedom, but it should be noted that the estimates of α and β were very highly correlated with an estimated value of -0.97 . Residuals were also highly skewed, nevertheless, for comparison with later results parameter estimates are presented in Table 3.

Since the 63 sets of observations came from 34 sets of studies, it seems sensible to allow that replicates from one study may not be typical of other studies. A multilevel model therefore offers a promising approach. Study specific estimates of the three model parameters α , β and κ can be obtained, but they are assumed to come from the same population of ocean systems. Borsuk *et al.* (2001) assumed normal priors for the system specific parameters with means drawn from system specific uniform hyperpriors but the variances were drawn from population level inverse gamma priors. It should be noted that these models are quite sensitive to prior choice, but the results presented here correspond to those used in the original publication. However, one feature of this meta-analysis was the assumption of log-normal residuals, which explains much of the difference between the non-linear least squares parameter estimates presented in Table 3 and the single level Bayesian posterior estimates presented in the first column of Table 4.

| | Estimate | S.E. | t value | $Pr(> t)$ |
|----------|----------|--------|---------|-------------|
| α | 0.4796 | 0.1175 | 4.081 | 0.0001 |
| β | 0.9324 | 0.0684 | 13.630 | 0.0000 |
| κ | 0.0007 | 0.0001 | 3.610 | 0.0006 |

Table 3: Conventional non-linear regression applied to Borsuk data

| | Single level | Multilevel |
|----------|---------------------|---------------------|
| | Posterior Mean (sd) | Posterior mean (sd) |
| α | 0.6126(0.0666) | 0.8166(0.0649) |
| β | 0.4103(0.0316) | 0.3989(0.0069) |
| κ | 0.0012(0.0003) | 0.0077(0.0023) |

Table 4: Posterior estimates from Bayesian models fitted to the BOD data

Table 4 gives the posterior mean of the upper level parameters. It can be seen that $\hat{\kappa}$ is considerably larger in the multilevel model, and that the values of $\hat{\alpha}$ and $\hat{\beta}$ have also altered somewhat. However, knowing these global parameters is of little interest when studying a particular estuarine system. What is of more interest are the estimates for individual systems. These are presented in Figure 3 which denotes the posterior median by a dot and the posterior 95% credible interval by an arrow. It can be seen that the individual system posterior distributions of β are very similar, posterior distributions of α vary more but overlap considerably and it only the posterior distributions of κ that appear to diverge to any great extent.

In deciding on the value of the system specific intercepts, it is of considerable interest to compare the predictions from the non-linear least squares fit and the Bayesian multi-level model. Figure 4 plots predicted benthic oxygen demand against recorded benthic oxygen demand for these two models. This demonstration serves to illustrate the potential for multilevel non-linear modelling since these applications are particularly common in many of the natural sciences. It is clear that predictions from the multilevel model are more faithful to the observed BOD than the non-linear least squares model.

5. Multivariate modelling

Finally, the ubiquity of multivariate data in environmental applications means that models which can account for dependencies in the data are of particular interest. The previous example demonstrated the potential to shrink estimates within nested layers of a model. Multivariate hierarchical models offer the additional potential to shrink parameter estimates across variables, reducing the associated uncertainty. Such an approach has been demonstrated in respect of road traffic casualties (Bailey and Hewson, 2004). Assuming the expected casualty count \mathbf{o} (offset), the observed casualty count \mathbf{Y} can be modelled as $\mathbf{Y} \sim \text{Poisson}(\mathbf{o}\boldsymbol{\lambda})$ where $\log(\boldsymbol{\lambda}) = \boldsymbol{\beta} + \boldsymbol{\nu}$. $\boldsymbol{\nu}$ is assumed multivariate normal with zero mean and unknown covariance matrix. An illustration of the potential shrinkage obtained from this model is given in figure 5. By superimposing summaries of the posterior distribution of the ranks from both the independently fitted univariate models as well as the multivariate model it is apparent that these have been shrunk considerably by the use of the multivariate model.

It should be noted that it is possible to reformulate this model as a latent structure model given by:

$$\log \boldsymbol{\lambda} = \boldsymbol{\Delta} \boldsymbol{\Phi} + \boldsymbol{\zeta}$$

Where $\boldsymbol{\Phi}$ are assumed standard normal latent variables (in this case four independent latent variables were required), $\boldsymbol{\zeta}$ are variable specific variances (zero mean and unknown

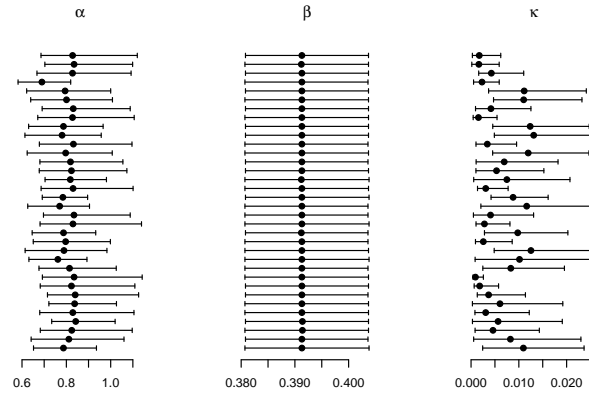


Figure 3: Posterior median (dots) and 95% credible intervals for estuarine system specific estimates of the three BOD model parameters α , β and κ

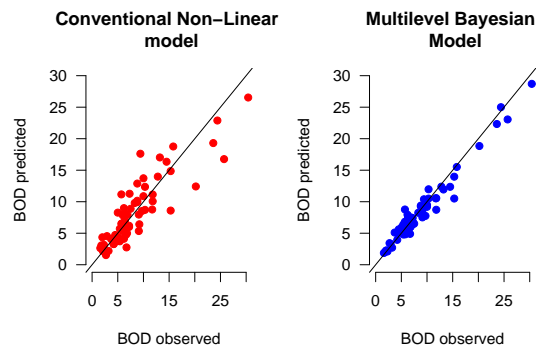


Figure 4: Comparison of predictions from non-multilevel non-linear least squares model and from multilevel Bayesian model fitted to BOD data

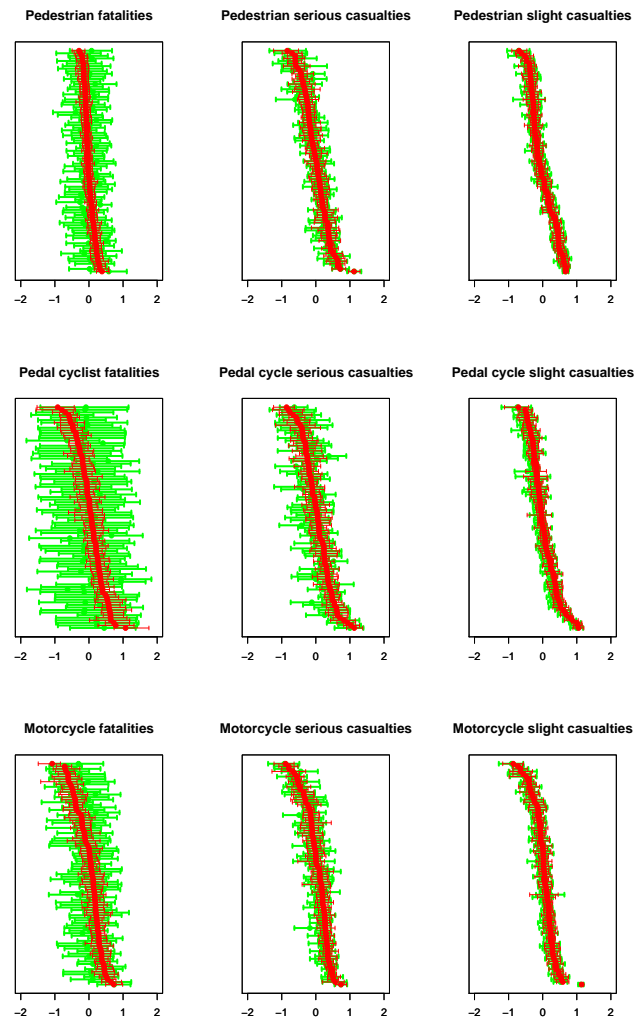


Figure 5: Shrinking random effects (intercepts) by borrowing strength across variables: Multivariate model (red) superimposed on nine univariate models (green)

variance). To ensure identifiability, Δ was a matrix of loading variables assumed zero above the diagonal, *Normal*(1, 1) below the diagonal and *Normal*(1,1) but constrained to be positive on the diagonal. Using such a model, the DIC was estimated as 4917.780, comparable to that obtained from a log-multivariate normal model illustrated above where the DIC was estimated as 4986.4. This latent structure approach offers considerable potential for expansion to deal with substantive problems arising in environmental studies. One extension is to model spatial data, for example areal data can be modelled by superimposing a conditional auto-regressive structure on a single latent variable (Christensen and Amemiya, 2003; Wang and Wall, 2003) or by using a latent variable for kriging purposes Minozzo and Fruttini (2004). Latent structure models also have been extended to deal with longitudinal data (Dunson and Herring, 2005) and there are proposals for dynamic latent structure models in Dunson (2003). One multilevel application enjoying high levels of interest concerns attempts to determine an association between PM10 levels and subsequent elevations in mortality, having controlled for a number of known meteorological confounding factors (Dominici *et al.*, 2000). This work uses additive smoothing models. Recent attempts to improve the strength of inference has considered bivariate models (Dominici *et al.*, 2004), but requires development to include random effects in a multivariate setting. Therefore, proposals to use Dynamic GLMs (Chiogna and Gaetan, 2002) are of interest, as these could be combined with dynamic latent structure models to this end, so that estimators for different mortality types could be shrunk in accordance with their dependence structure.

6. Conclusion

This paper has provided a very brief review of the role of multilevel models within environmental applications. They are established as an important tool for modelling observational data especially where it has been collected in an uncontrolled and unbalanced way. Multilevel models have also seen regular use in meta-analysis and similar shrinkage applications where one seeks to model an individual phenomena in relation to a “population” of similar observations. It seems likely that multilevel models are currently enjoying growing interest in their potential to deal with multivariate analysis in many contemporary environmental applications.

References

- Aitkin M., Anderson D. and Hinde J. (1981) Statistical Modelling of Data on Teaching Styles, *J. R. Statist. Soc. A*, 144, 419–461.
- Bailey T.C. and Hewson P.J. (2004) Simultaneous modelling of multiple traffic safety performance indicators using a multivariate generalised linear mixed model, *J. R. Statist. Soc. A*, 167, 3, 551–567.
- Bates D. and Watts D. (1988) *Nonlinear Regression Analysis and Its Applications*, Wiley, NY.
- Borsuk M., Higdon D., Stow C. and Reckhow K. (2001) A Bayesian hierarchical model to predict benthic oxygen demand from organic matter loading in estuaries and coastal zones, *Ecological Modelling*, 143, 3, 165–181.
- Boslaugh S., Luke D., Brownson R., Naleid K. and Kreuter M. (2004) Perceptions of

- neighbourhood environment for physical activity: Is it “who you are” or “where you live”?, *Journal of Urban Health*, 81, 4, 671–681.
- Browne W. and Draper D. (2000) Implementation and performance issues in the Bayesian and likelihood fitting of multilevel models, *Computational Statistics*, 15, 391–420.
- Chiogna M. and Gaetan C. (2002) Dynamic generalized linear models with application to environmental epidemiology, *Appl. Stat.*, 51, 4, 453–468.
- Christensen W.F. and Amemiya Y. (2003) Modeling and prediction for multivariate spatial factor analysis, *Journal of Statistical Planning and Inference*, 115, 543–564.
- Congdon P. (2001) *Bayesian Statistical Modelling*, Wiley, Chichester.
- Coull B., Menzetti M. and Ryan L. (2003) A Bayesian hierarchical model for risk assessment of methylmercury, *Journal of Agricultural, Biological and Environmental Statistics*, 8, 3, 253–270.
- Dominici F., Samet J. and Zeger S. (2000) Combining evidence on air pollution and daily mortality from the 20 largest us cities: a hierarchical modelling strategy, *Journal of the Royal Statistical Society Series A*, 163, 263–284.
- Dominici F., Zanobetti A., Zeger S.L., Schwarz J. and Samet J.M. (2004) Hierarchical bivariate time series models: a combined analysis of the effects of particulate matter on morbidity and mortality, *Biostatistics*, 5, 3, 341–360.
- Dunson D. (2003) Dynamic Latent Trait Models for Multidimensional Longitudinal Data, *J. Am. Statist. Ass.*, 98, 555–563.
- Dunson D. and Herring A. (2005) Bayesian latent variables for mixed discrete outcomes, *Biostatistics*, 6, 11–25.
- Gelman A. (2005) Hierarchical (multilevel) modeling: what it can and can’t do.
- Goldstein H. (1989) Restricted Unbiased Iterative Generalised Least Squares Estimation, *Biometrika*, 76, 622–623.
- Goldstein H. (1991) Nonlinear Multilevel Models With an Application to Discrete Response Data, *Biometrika*, 78, 45–51.
- Goldstein H. (2003) *Multilevel Models (3rd Edition)*, Arnold, London.
- Goldstein H. and Rasbash J. (1992) Efficient computational procedures for the estimation of parameters in multilevel models based on iterative generalised least squares, *Computational Statistics and Data Analysis*, 13, 63–71.
- Guerin D., Crete J. and Mercier J. (2001) A Multilevel Analysis of the Determinants of Recycling Behavior in the European Countries, *Social Science Research*, 30, 195–218.
- Hinde J. (1982) Compound Poisson regression models, in: *GLIM 82*, Gilchrist R., ed., Springer-Verlag, New York, 109–121.
- Hodges J. (1998) Some algebra and geometry for hierarchical models, applied to diagnostics, *J. R. Statist. Soc. B*, 60, 497–521.
- Hox J. (2002) *Multilevel Analysis: Techniques and Applications*, Erlbaum, New Jersey.
- Langford I., Bentham G. and McDonald A. (1998) Multilevel modelling of geographically aggregated health data: a case study on malignant melanoma mortality and uv exposure in the european community, *Statistics in Medicine*, 17, 41–58.
- Lawless G.F. (1987) Negative binomial and mixed Poisson regression, *The Canadian Journal of Statistics*, 15, 209–225.
- Lee Y. and Nelder J. (1996) Hierarchical Generalized Linear Models, *J. R. Statist. Soc. B*, 58, 619–678.
- Lindstrom M. and Bates D. (1988) Newton-Raphson and EM Algorithms for Linear Mixed-Effects Models for Repeated-Measure Data, *J. Am. Statist. Ass.*, 83, 1014–1022.
- Minozzo M. and Fruttini D. (2004) Loglinear spatial factor analysis: an application to

- diabetes mellitus complications, *Environmetrics*, 15, 423–434.
- Nelder J. and Wedderburn R. (1972) Generalized Linear Models, *J. R. Statist. Soc. A*, 135, 370–384.
- Pinheiro J., Bates D., DebRoy S. and Sarkar D. (2004) *nlme: Linear and nonlinear mixed effects models*.
- Pinheiro J.C. and Bates D.M. (2000) *Mixed-Effects Models in S and S-PLUS*, Springer, New York.
- R Development Core Team (2004) *R: A language and environment for statistical computing*, R Foundation for Statistical Computing, Vienna, Austria, ISBN 3-900051-00-3.
- Robinson W. (1950) Ecological Correlations and the Behavior of Individuals, *American Sociological Review*, 13, 3, 351–357.
- Schall R. (1991) Estimation in generalized linear models with random effects, *Biometrika*, 78, 719–727.
- Snijders T. and Bosker R. (1999) *Multilevel Analysis*, Sage, London.
- Spiegelhalter D., Thomas A. and Best N. (1998) WinBUGS version 1.1.1 user manual, Technical report, MRC Biostatistics Unit, Cambridge.
- Spiegelhalter D.J., Best N.G., Carlin B.P. and van der Linde A. (2002) Bayesian measures of complexity and fit, *J. R. Statist. Soc. B*, 64, 583–639.
- van Lenthe F., Brug J. and Mackenbach J. (2005) Neighbourhood inequalities in physical inactivity: the role of neighbourhood attractiveness, proximity to local facilities and safety in the Netherlands, *Social Science and Medicine*, 60, 4, 763–775.
- Venables W.N. and Ripley B.D. (2002) *Modern Applied Statistics with S*, Springer, New York, fourth edition.
- Wang F. and Wall M.M. (2003) Generalized common spatial factor model, *Biostatistics*, 4, 569–582.
- Wold B., Torsheim T., Currie C. and Roberts C. (2004) National and school policies on restrictions of teacher smoking: a multilevel analysis of student exposure to teacher smoking in seven European countries, *Health Education Research*, 19, 3, 217–226.